Metody Statystyczne 2

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- przypomnienie
- zmiana zmiennych losowych 1D
- wielowymiarowe zmienne losowe
- zmiana zmiennych losowych w 2D

$$P(a < y \le b) = \int_a^b f(u)\,du$$

$$\hookrightarrow \int_{u_{max}}^{u_{max}} f(u)\,du = \underline{1}$$

$$P(x < y \le x + dx) = f(x)\,dx \geq 0$$

$$\hookrightarrow \forall_u\, f(u) \geq 0$$

wartość oczekiwana:
(expectation value)

$$E(X) = \int_{-\infty}^{+\infty} u\, f(u)\, du$$

$$E(g(X)) = \int_{-\infty}^{+\infty} g(u)\, f(u)\, du$$

$g(x) = c$ ← pewna stała

$$E(g(X)) = \int_{-\infty}^{+\infty} c\, f(u)\, du = c \underbrace{\int_{-\infty}^{+\infty} f(u)\, du}_{1} = c$$

$g(x) = ax + b$

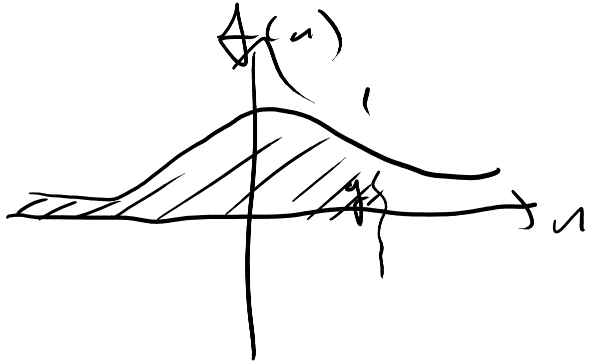
$$E(aX+b) = \int_{-\infty}^{+\infty} (au+b)\, f(u)\, du =$$

$$= \left(\int_{-\infty}^{+\infty} au\, f(u)\, du\right) + \left(\int_{-\infty}^{+\infty} b\, f(u)\, du\right) =$$

$$= a \underbrace{\int_{-\infty}^{+\infty} u\, f(u)\, du}_{E(X)} + b \underbrace{\int_{-\infty}^{+\infty} f(u)\, du}_{1} = aE(X) + b$$

dystrybuanta

$$F(y) = \int_{-\infty}^{y} f(u)\, du$$



$$\mathrm{var}(X) = \int_{-\infty}^{+\infty} du \left(u - E(X)\right)^2 f(u) =$$

$$= \int_{-\infty}^{+\infty} du \left(u^2 - 2uE(X) + E^2(X)\right) f(u) =$$

$$= E(X^2) + \left(\int du \left(-2uE(X)\right) f(u)\right) + E^2(X) =$$

$$= E(X^2) - 2E^2(X) + E(X) = E(X^2) - E^2(X)$$

$$\sigma(X) = \sqrt{\mathrm{var}(X)}$$

$$\sigma(g(X)) = \sqrt{\mathrm{var}(g(X))}$$

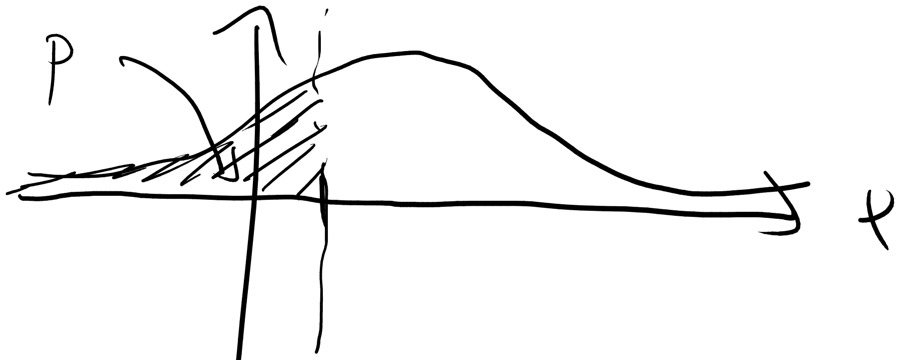
$$\mu_k(X) = E\left((X - E(X))^k\right)$$

$$\mu_2(X) = E\left((X - E(X))^2\right)$$

kwantyl

x_p jest kwantylem rzędu p gdy

$$F(x_p) = p$$



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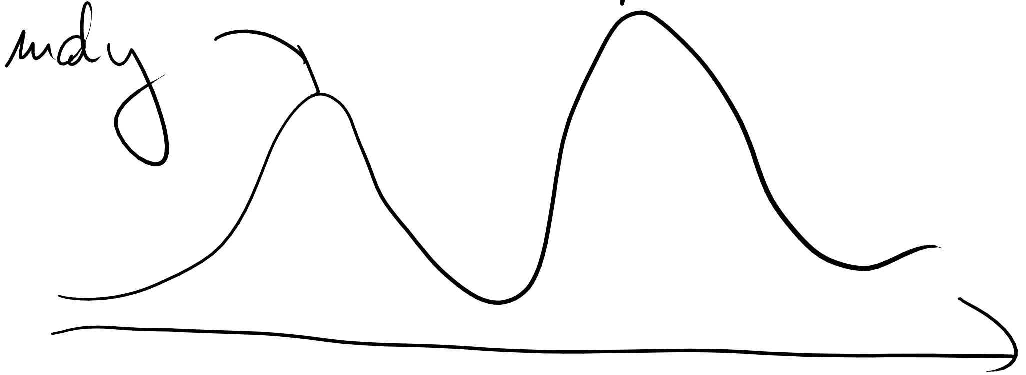
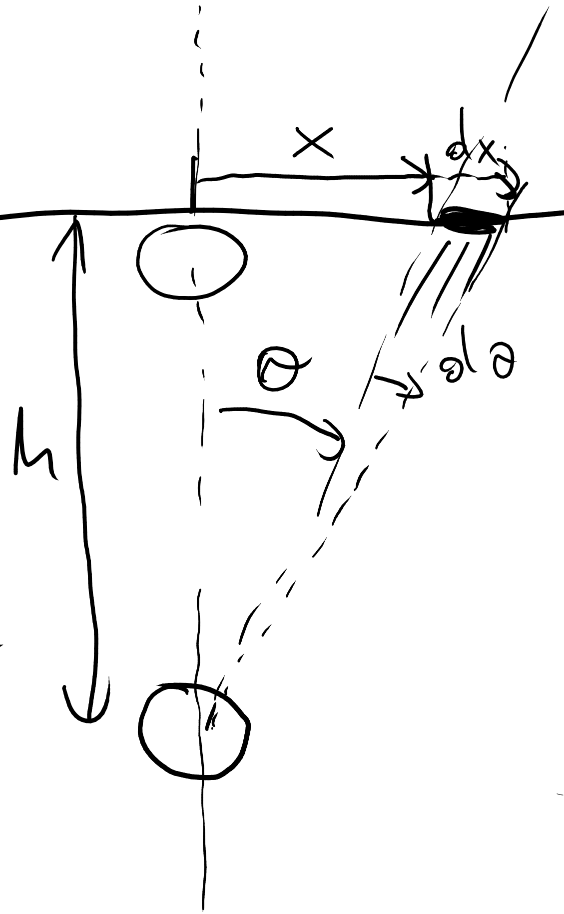




$d\theta, dx$ – имотеч

$$f(\theta)\,d\theta = g(x)\,dx$$

$$f(\theta)\,d\theta = g(x)\left|\frac{dx}{d\theta}\right| d\theta$$

$$f(\theta) = g(x)\left|\frac{dx}{d\theta}\right|$$

$$f(\theta(x)) = g(x)\left|\frac{dx}{d\theta}\right|$$

$$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$f(\theta) = \frac{1}{\pi}$$

$$\Downarrow$$

$$g(x)$$

$$\tan(\theta) = \frac{x}{h}$$

$$x = x(\theta) = h \tan(\theta)$$

$$\theta = \theta(x) = \arctan\left(\frac{x}{h}\right)$$

$$\frac{dx}{d\theta} = h \left(\frac{1}{\cos\theta}\right)^2 > 0$$

$$f(\theta)\, d\theta = g(x)\, h \left(\frac{1}{\cos^2\theta}\right) d\theta$$

$$\frac{1}{\pi} = f(\theta)$$

$$g(x) = \frac{1}{\pi} \frac{\cos^2\theta}{h} = \frac{1}{\pi} \frac{\cos^2\left(\arctan\left(\frac{x}{h}\right)\right)}{h} = \dots$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \frac{1}{\cos^2\theta}$$

$$\cos^2\theta = \frac{1}{1+\tan^2(\theta)}$$

$$\dots = \frac{1}{h^2\pi + \pi x^2} = g(x)$$

rozkład Cauchiego

$$\int_{-\infty}^{+\infty} dx\, g(x) \stackrel{?}{=} 1$$

var(X)

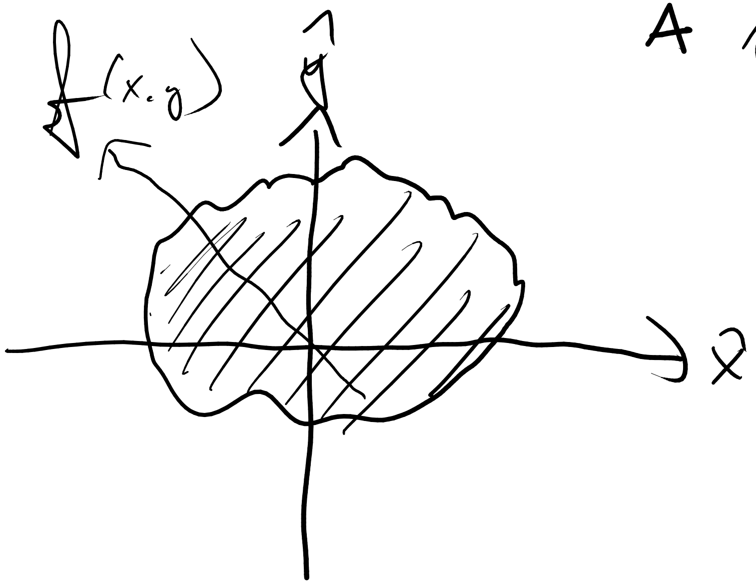
wielowymiarowe zmienne losowe

(x, y) $\quad x, y \in \mathbb{R}$

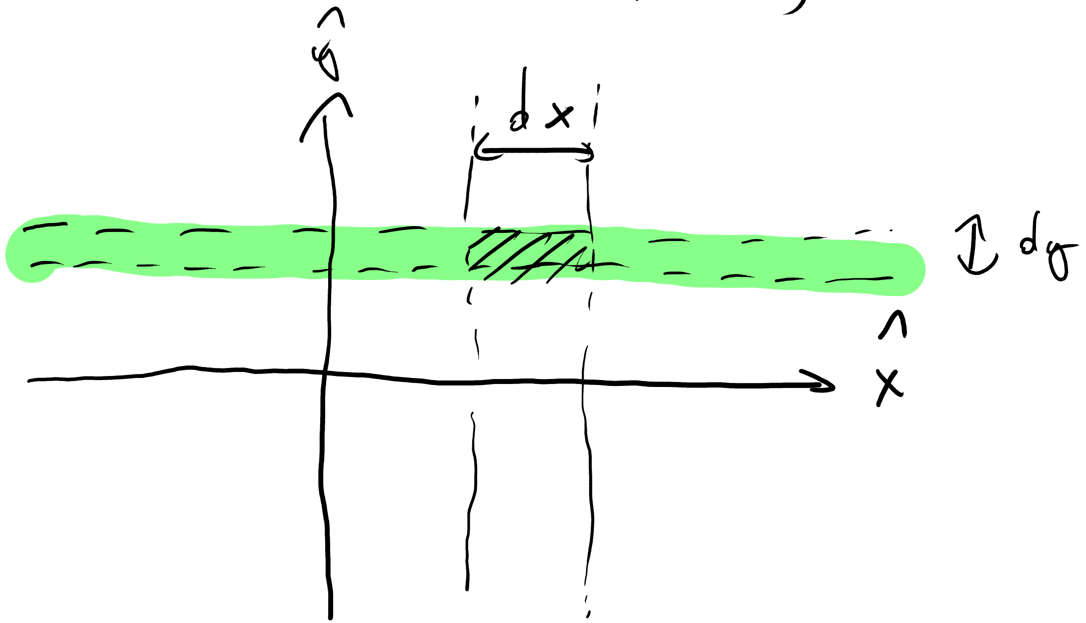
$(x, y, z), (x, y, z, u),$
$(x, y, z, u, w) \ldots$

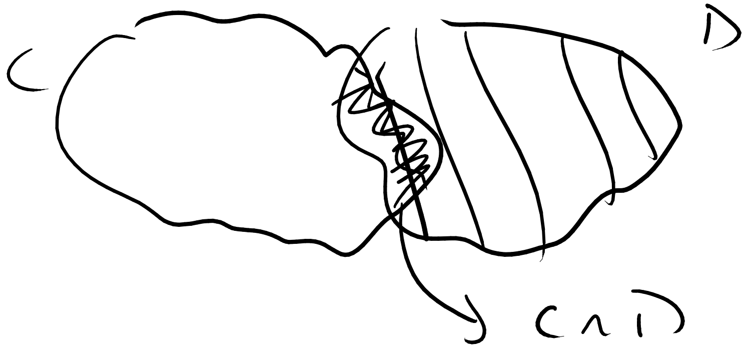
$$P(r \in A) = \iint_A f(x,y)\,dx\,dy$$

funkcja gęstości prawdopodobieństwa



$$P(C|D) = \frac{P(C \wedge D)}{P(D)}$$





$$f(x|y)\,dx = \frac{f(x,y)\,dx\,dy}{f(y)\,dy}$$

$$f(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx$$

$$\mathrm{cov}(X,Y) = \mathbb{E}\big((X-\mathbb{E}(X))(Y-\mathbb{E}(Y))\big) =$$

$$= \iint dx\,dy\,(x-\mathbb{E}(x))(y-\mathbb{E}(y))\,f(x,y)$$

$$\mathrm{cov}(X,X) = \mathrm{var}(X) \qquad \mathrm{cov}(X,Y) = \mathrm{cov}(Y,X)$$

$$\mathrm{cov}(Y,Y) = \mathrm{var}(Y)$$

$$K_{X,Y} = \begin{pmatrix} \mathrm{var}(X) & \mathrm{cov}(X,Y) \\ \mathrm{cov}(Y,X) & \mathrm{var}(Y) \end{pmatrix}$$

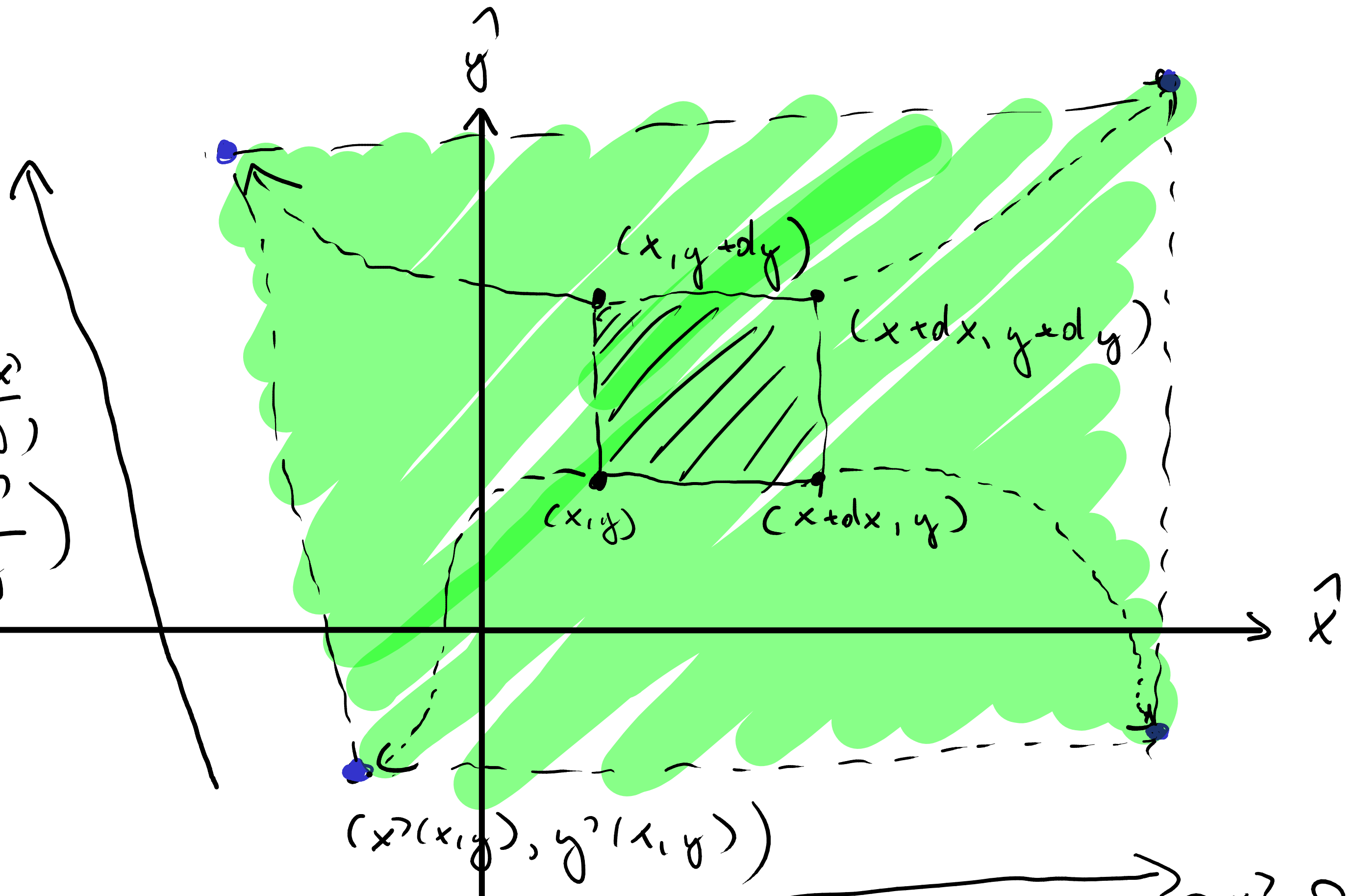
$$\rho_{X,Y} = \mathrm{corr}(X,Y) = \frac{\mathrm{cov}(X,Y)}{\mathrm{var}(X)\,\mathrm{var}(Y)}$$

$$\iint f(x,y)\,dx\,dy$$

$$\begin{cases} x' = x'(x,y) \\ y' = y'(x,y) \end{cases}$$

$$g(x',y') = \,?$$

$$dy\left(\frac{\partial x'}{\partial y}, \frac{\partial y'}{\partial y}\right) = \vec{a}$$



$$\iint f(x,y)\,dx\,dy =$$

$$= g(x',y') \cdot \langle \text{powierzchnia} \rangle$$

$$dx\cdot\left(\frac{\partial x'}{\partial x}, \frac{\partial y'}{\partial y}\right) = \vec{b}$$

$$x'(x+dx, y) = x'(x,y) + dx \cdot \frac{\partial x'}{\partial x}$$

$$y'(x+dx, y) = y'(x,y) + dx \frac{\partial y'}{\partial x}$$

$$x'(x, y+dy) = x'(x,y) + dy \frac{\partial x'}{\partial y}$$

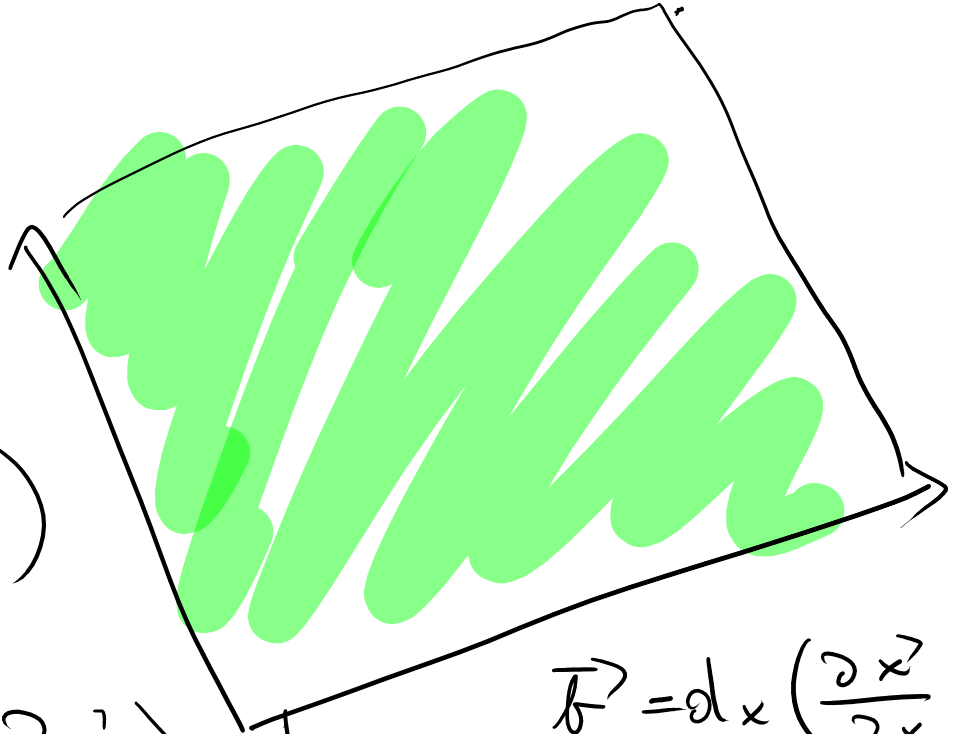
$$y'(x, y+dy) = y'(x,y) + dy \frac{\partial y'}{\partial y}$$

$$x'(x+dx, y+dy) = x'(x,y) + dx \frac{\partial x'}{\partial x} + dy \frac{\partial y'}{\partial y}$$

$$y'(x+dx, y+dy) = y'(x,y) + dx \frac{\partial y'}{\partial x} + dy \frac{\partial y'}{\partial y}$$

wolumn(x, y)

$$\vec{a} = dy\left(\frac{\partial x'}{\partial y}, \frac{\partial y'}{\partial y}\right)$$



$$\vec{b} = dx\left(\frac{\partial x'}{\partial x}, \frac{\partial y'}{\partial x}\right)$$

$$dx\,dy\left|\det\begin{pmatrix} \frac{\partial x'}{\partial x} & \frac{\partial y'}{\partial y} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} \end{pmatrix}\right| = dx\,dy\,|\det(J)|$$

$$\int f(x,y)\,\cancel{dx\,dy} = g(x',y') \cdot \langle \text{powierchnie} \rangle$$

$$= \underbrace{g(x',y')}_{}\,|\det(J)|\,\cancel{dx\,dy}$$