przestrzeń zdarzeń losowych

$$(\Omega, F, P)$$

Ω – zdarzeń elem. F – funkcja przypisująca pr.

$$\Omega = \{ \boxdot, \boxdot\!\boxdot, \boxdot\!\boxdot\!\boxdot, \boxdot\!\boxdot, \boxdot\!\boxdot, \boxdot\!\boxdot\!\boxdot \}$$

$$F = \{ \emptyset, \{ \boxdot, \boxdot\!\boxdot\!\boxdot, \boxdot\!\boxdot \}, \dots, \Omega \}$$

$P(\{\boxdot\}) = \frac{1}{6}$ $P(\{\boxdot\!\boxdot\}) = \frac{1}{6}$

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$P(\emptyset) = ?$

$P(\{\boxdot, \boxdot\!\boxdot\!\boxdot, \boxdot\!\boxdot\}) = ?$

Kolmogorov 1933

$$P(E) \geq 0 \quad \forall E \in F$$

$$P(\Omega) = 1$$

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$ jeżeli E_i parami rozłączne

$A^c = \Omega \setminus A$

$$P(A) + P(A^c) = P(A \cup A^c) = P(\Omega) = 1$$

$$P(A^c) = 1 - P(A)$$

$$P(\Omega^c) = 1 - P(\Omega) = 0 = P(\emptyset)$$

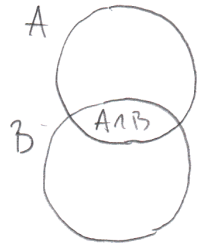


$$P(A \cup B) = P(A) + P(B \setminus (A \cap B))$$

$$P(B) = P(B \setminus (A \cap B)) + P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

[wikipedia, ...] $\begin{cases} A \in B \rightarrow P(A) < P(B) \\ \forall B \in F: \; 0 \leq P(B) \leq 1 \\ \dots \end{cases}$



$$P(A \mid B) \equiv \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A \mid B)\, P(B)$$

$$P(B \cap A) = P(B \mid A)\, P(A)$$

$$P(A \mid B) = P(A) \cdot \frac{P(B \mid A)}{P(B)}$$

Bayes

hipoteza

$$P(H \mid D) = P(H) \cdot \frac{P(D \mid H)}{P(D)}$$

dane

mieszamy $P(\cdot)$ oraz $P(\cdot \mid \cdot)$...

$$P(A \cap B \mid X) = \frac{P(A \cap B \cap X)}{P(B \cap X)} \frac{P(B \cap X)}{P(X)} = P(A \mid B \cap X)\, P(B \mid X)$$

$$P(A \cap B \mid X) = P(B \cap A \mid X)$$

$$P(A \mid B \cap X)\, P(B \mid X) = P(B \mid A \cap X)\, P(A \mid X)$$

$$P(A \mid B \cap X) = P(A \mid X) \frac{P(B \mid A \cap X)}{P(B \mid X)}$$

Łatwo $P(\cdot \mid \cdot)$!

hipoteza — "ukryte" założenia

$$P(H|D\wedge X) = P(H|X)\,\frac{P(D|A\wedge X)}{P(D|X)}$$

dane

$$P(A\vee B|X) = P(A|B\wedge X)\,P(B|X) = \frac{P(A\wedge B\wedge X)}{P(B\wedge X)}\cdot\frac{P(B\wedge X)}{P(X)}$$

$$= P(B|A\wedge X)\,P(A|X)$$

$$P(A\vee B|X) = P(A|X) + P(B|X) - P(A\wedge B|X)$$

... można w prostszych rachunkach $P(\cdot)\to P(\cdot|\cdot)$

$\cap \to \wedge$

$\cup \to \vee$

Kołmogorow $\to$ Cox R.T.

$$P(a\wedge b|x) = P(a|b\wedge x)\,P(b|x) = P(b|a\wedge x)\,P(a|x)$$

$$P(a\vee b|x) = P(a|x) + P(b|x) - P(a\wedge b|x)$$

propozycje logiczne

$A\wedge B \equiv AB$

$A\vee B \equiv A+B$

likelihood

$$P(H|D_2 D_1 X) = P(H|D_1 X)\cdot\frac{P(D_2|H D_1 X)}{P(D_2|D_1 X)}$$

X' X' X' X'

posterior prior